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Basic Theory of Ultrasonic Scattering by Defects: Numerical Studies and Features for Experimental Application

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Abstract
Well, the theory of scattering of elastic waves is difficult. I don't want to minimize that, but I thought I might make a few diversionary introductory remarks. My first reference will be the Los Angeles Times of today, the astrology column. It advises those born under the sign of Leo to choose their words carefully to avoid trouble. That happens to be applicable to yours truly. I hope I do it.

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BASIC THEORY OF ULTRASONIC SCATTERING BY DEFECTS: NUMERICAL STUDIES & FEATURES FOR EXPERIMENTAL APPLICATION*†

J. A. Krumhansl
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Ithaca, New York

Well, the theory of scattering of elastic waves is difficult. I don't want to minimize that, but I thought I might make a few diversionary introductory remarks. My first reference will be the Los Angeles Times of today, the astrology column. It advises those born under the sign of Leo to choose their words carefully to avoid trouble. That happens to be applicable to yours truly. I hope I do it.

The second technique which has been found useful in the unruly audiences at Cornell is to show your last slide first or you'll never get there. So, that's what I want to do. Figure 1 (see Fig. 1 of the Appendix) is a three-dimensional plot of elastic wave cross sections, which we have calculated from 0 to 180° as a function of ka. This is for a longitudinal plane wave incident on a spherical cavity in titanium, and this is the differential scattering cross section for the transverse component. The calculation is for all ka from 0 to 6. The experimentalist can, for example, if he prescribes the frequency spectrum, which is, of course, a range of ka, take a cut across this with his computer and estimate the total difference across section for a transversely polarized receiving transducer, with this incident longitudinal wave. We're cost conscious at the university. This data presentation costs about a dollar and took three minutes.

Now, given the end of the story, I don't know whether anybody is interested in my filling in the details, but that's what I'm here for. So, let me proceed. We're doing this in the solid state theory group at Cornell. The people listed in the Appendix are the participants. We began in September 1974. Jim Gubernatis really deserves special mention for giving technical leadership to the program and seeing that it moved, and particularly for supervising a number of the computations. Without this help I don't think we would have gotten where we are. The rest of us are active participants. One last bit of perspective: the cost of this total program for this year is about that of two Tektronix scopes with ordinary software.

It's an interesting academic experiment. Can some denizens of a university theoretical physics group interact with engineers? We've had a good time, and also learned a lot by interactions with Prof. Pao in the Cornell Mechanics Department during the course of the past year.

We tried to look around and see where we might make a contribution in the overall NDE program and the contribution we chose to make was to update the theory of scattering, and we found that it was sort of a cold water shock when you move from the simplicity of quantum mechanics with its scalar fields

* To supplement this paper which was presented at the symposium, a technical summary article, "Theory of the Scattering of Ultrasound by Flaws", by J. E. Gubernatis, E. Domany, M. Huberman, and J. A. Krumhansl, is incorporated as an Appendix.

† Research sponsored by ARPA/AFML Center for Advanced NDE.

into the tensor fields of elasticity. That's part of the problem; many theoretical solid state physicists who have written many papers on band theory tend to underestimate seriously the difficulty of this problem. I don't want to make this sound so great, but the problem is just damn tough.

What has happened in this subject? Well, the last significant work, as you heard referred to, is that by Truell and his collaborators, and I think we all should feel regret about the untimely death of Rohn Truell who really was much ahead of his time in this subject. During the past ten years, there have been significant advances in the hardware of formal scattering theory and the use of computers, and that's what we've tried to apply. The kind of investigation we're doing is somewhat complementary to that reported by Dick Cohen.

Let me just emphasize that it really is difficult to get exact solutions, and the business of doing exact solutions for a spheroid in a partial wave basis is going to be a tour de force, as Pao will also attest to. What we're trying to do is see whether we can use what has happened in other branches of physics that occasionally substitute the vast power of computers for formalism, admittedly exploratory and nonrigorous, but at least giving guidance. In fact, Fig. 1 is a comparison between an exact calculation using these methods that have been referred to, and an approximate calculation. Our present objective is a feasibility study of these approximate calculations. The present situation is that we have checked our theory, we've checked computer codes, we've made application to standard geometry scatterers in aluminum, titanium and stainless steel, including spherical cavities or imbedded spheres, and we've looked at both longitudinal and transverse incident waves and mode conversion.

Let me give you a technical overview for perspective. The physics contains constitutive relations, elastic relations, dynamics, Newton's laws, and the equations of motion. There are two philosophies of solving such problems, one very much more familiar than the other. The differential equation technique combines these ingredients to give you wave equation: you do polynomial expansion, you do boundary matching and you do partial wave scattering analyses and compute the scattered fields.

But there's another way, which is to invert, as I'll show, into a form of an integral equation. One of the big advantages of this formulation is, first of all--at least to a computer (not for a pencil pusher)--is that iteration techniques are possible, which greatly aids what one can do. Secondly, it's possible to get very economical expressions for general scattering cross sections and conservation laws, which are important to physicists and are sometimes useful as engineering checks.

Now, again for perspective, and to finally define the present frame of reference, there are two regimes: the defraction limit where ka is very much less than one and the geometric optics limit where ka is very much greater than one. In the geometric optics limit, one has ray theory effects, edge and surface waves. Seismologists are thoroughly familiar with these areas. Our box has to do with the opposite scattering regime for which Bruce Thompson set up a conceptual applications framework.

Here's what we've done in our applications to date. We've programmed the exact results for a sphere, using a partial wave basis. Then we start exploring their various approximate techniques. In principle, though we haven't done this, we believe the approximate techniques can be extended to complex shape flaws, cubes, holes, etc. That's the background.

The quantities which we need are the energy flux terms. The energy flux is, simply, the stress components times the velocity. Stress times velocity is the rate of flow of energy. So, any theory must give you asymptotically the σ_{ij} and the particle velocities. And that's what we point our computer at and our formulas at. We can either get a computer total cross section by integrating the flux over any bounding surface or computer partial differential cross section whose evaluation is detailed in the Appendix. The "flow" of the problem is that there's an incident flux, there are fields at the scatterer, and then you carry out asymptotic approximations and find out what goes on at far distances.

The physics: as indicated previously, there's a displacement field, there's a strain field, the comma being the standard for partial differential of μ_k with respect to l. There is a stress field, which is related to the strain field by a local modulus. This yields a differential operator on displacements u equated to forces f.

Not, let me just interrupt your thinking about this hardware and consider the perversity of the way we've learned our laws of nature. What we want to know is u, but in fact, we have an equation which gives us the f's in terms of the u, not the other way around. But what you ordinarily do is apply forces to a system and then measure the displacements or the scattered strain fields or what have you. So, there's a certain perverseness about the way we learn our physics, but that's the way it is.

Now, if you want to get around that, Mr. Green said that what you need, then, is a Green's function. And the Green's function is simply a response function that states the problem the other way around, given the f's you get the u from something which is a transfer function or a response function. If you wish, this may be regarded a generalization of the response function or transfer function for an amplifier. Now, a field problem is the same thing but with a multichannel system, an infinite number of channels for each point in space, relating forces applied, displacements out.

Now, let's look what happens in--what I'm trying to do is motivate for you the integral equation formulation of this problem. Well, first I have a transducer and if it were a perfect medium, out would come u^0, which is the incident wave. But now there's a flaw in this material, That flaw feeds back because it produces a local stress. That local stress combines with the stress imposed by the transducer and feed back out and so now you have an infinite number of channel parallel feedback amplifier systems, whose equations you can write down as indicated in the Appendix, and what one has is an integral equation.

Now, one can probably solve no greater number of integral equations than one can do exactly by the partial wave differential analysis.

But, you can carry out systematic approximations more easily on integral equations. What's the cheapest thing you can do? (See Eqn. 1 of the Appendix). The cheapest thing you can do is replace that general u by u^o to begin with and that begins a hierarchy of iterations. And the iterative technique is something that the computer just loves. That's the basis of what we've done or what we're seeking to do. We're thus seeking to go beyond the limitations of the formal solutions.

This first iteration in the language of quantum mechanical scattering is called the "first Born approximation", this is what we have tested extensively. Now, going beyond that the situation is relatively straightforward in principle. Given the need and $100 instead of $1, you can do a calculation for a nonsimple shape much more exactly.

Now, I've almost told you the whole story as regards motivation and the setup of the problem. Ideally, in the long run, one would have a handbook of sample solutions to stick in as improvements beyond the first Born approximation; this probably is needed to handle scattering by a spheroid, edge, crack, or things of this sort. In the long wavelength limit, we can use the Eshelby solutions for the displacement field as a first step in iteration, and this was done by Mow and by Knopoff. At long wavelength this yields the exact scattering result in the first Born iteration. In other words, if you have any situation, this integral equation method provides an explicit receipe for what to do with that intuition as regards the scattered strain and stress fields. Then you can correct again, presuming always, which will worry a mathematician and sometimes the computer, that the iteration series converges and doesn't run wild.

In conclusion, let me remind you of the beginning of this session. I think Bruce properly pointed out that the story of this experimental field is that scattering calculations are not nearly as complete as the experimentalists would want. However, perhaps the outlook isn't quite as pessimistic as it need be and we hope that our approach, using brute force computation, plus physics and engineering intuition, can give moderate decent guidance to experimental interpretation.

THEORY OF THE SCATTERING OF ULTRASOUND BY FLAWS

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Appendix

An integral equation governing the scattering of ultrasound by an arbitrarily shaped flaw is presented, and features of the scattered displacement and stress fields are discussed for the case of a flaw embedded in an isotropic medium. Also discussed are differential cross sections for the scattered power. These cross sections for a spherical flaw (cavity and inclusion) are evaluated by an approximation analogous to the first Born approximation in quantum mechanical scattering. The results of the calculations are compared with exact results for scattering of ultrasound by spheres. The relevance of this comparison to NDE, i.e., flaw identification, is discussed.

Introduction

Ultrasonic methods are of great importance in the study of structural and elastic properties of materials, particularly for nondestructive evaluation (NDE). Beyond the experimental aspects of signal production and processing, the central physics and materials science question is that of specifying how flaws, that is, inhomogeneities, scatter ultrasonic waves. We address ourselves to the theoretical analysis of this question.

We adopt a particular viewpoint by formulating the problem in terms of an integral equation describing the scattering of ultrasound from the volume of the flaw. We use an integral equation because integral equation methods adapted from scattering theory in quantum mechanics are susceptible to a wider variety of practical approximation and computational methods than are the traditional partial differential equation methods. The choice of a volume formulation of the integral equation instead of the alternative of a surface formulation[1] is largely a matter of personal preference, but we believe the volume formulation incorporates the material inhomogeneity in a more direct manner.

The basic scattering problem we treat involves a material having elastic stiffness constants C_{ijkl} and density ρ in which there is a flaw defining a region of space R and having a sharp, smooth surface S. The material inside the flaw has elastic stiffness constants C'_{ijkl} and density ρ' different from those of the host material. For many cases of interest, $C'_{ijkl} = \rho' = 0$.

Power generated by some source is directed toward the flaw. The presence of the flaw, however, prevents the incident power from propagating unhindered, and the flaw selectively scatters part of the incident power into various directions. What is of interest is to measure the amount of power, relative to the incident power, scattered into a given direction. Such a measurement may serve to differentiate between flaw geometries and hence would be a useful NDE measurement. For example, if diffraction effects are ignored, one expects power incident normal to the plane of a large disc (a crack) to scatter more strongly backwards than forwards. Such a large ratio between forward and backward scattering is not expected if the flaw is a sphere (a pore).

The Scattering Equation

The physical quantity that measures the scattering effectiveness of a flaw is the "cross section" of the flaw. This quantity is related to, but not equal to, the effective geometrical cross-sectional area of the flaw. There are several kinds of cross sections; one type is the differential cross section. This cross section is defined at a distance far from the flaw as the time average of the power flux scattered into a given element of solid angle relative to the time average of the total incident power. Since the power associated with a stress field σ_{ij} and a displacement field u_i is $\sigma_{ij}\dot{u}_{ij}$ (the dot denotes time differentiation and the repeated subscript is summed), we must, in order to compute the differential cross section, be able to find the scattered displacement field and its associated stress field. We can show[2] that the total displacement field exactly satisfies the integral equation:

$$u_i(\underline{r}) = u_i^o(\underline{r}) + \delta\rho\omega^2 \int_R d\underline{r}' g_{im}(\underline{r},\underline{r}')\, u_m(\underline{r}')$$
$$\delta C_{jklm} \int_R d\underline{r}' g_{ij,k'}(\underline{r},\underline{r}')\, u_{l,m'}(\underline{r}') \qquad (1)$$

where u_i^o is the displacement field associated with the incident power,

$$\delta C_{ijkl} = C'_{ijkl} - C_{ijkl}, \qquad \delta\rho = \rho' - \rho, \qquad (2)$$

and g_{ij} is the Green's function (response function) satisfying

$$C_{ijkl} g_{km,jl} + \rho\omega^2 g_{im} + \delta_{im}\delta(\underline{r}-\underline{r}') = 0. \qquad (3)$$

The $,_i$ denotes differentiation with respect to x_i, and the time dependence of all fields is assumed to be $e^{-i\omega t}$. There are several noteworthy features of the integral equation. First, the equation is valid for a flaw of arbitrary density, elastic constants, and shape. A cavity has $C'_{ijkl} = \rho' = 0$. Second, the equation is valid both inside and outside the flaw. Third, the equation automatically insures the continuity of the displacement and normal stress across the surface S. Fourth, it is an exact equation.

For Eq. (1) to be useful, the Green's function must be found. For a flaw embedded in an infinite, isotropic, elastic medium, the Green's function is easily found:[1,2]

$$g_{ij}(\underline{r}-\underline{r}') = \frac{1}{4\pi\rho\omega^2}\left[\beta^2 \frac{e^{i\beta R}}{R}\delta_{ij} - \frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}\left(\frac{e^{i\alpha R}}{R} - \frac{e^{i\beta R}}{R}\right)\right], \qquad (4)$$

where

$$\alpha^2 = \frac{\rho\omega^2}{\lambda + 2\mu}, \quad \beta^2 = \frac{\rho\omega^2}{\mu}, \text{ and } R = |\underline{r} - \underline{r}'| \qquad (5)$$

with λ and μ being the Lamé parameters of the medium hosting the flaw. In what follows we assume that the host medium is infinite and isotropic.

The displacement in (1) can be written as

$$u_i = u_i^o + u_i^s. \qquad (6)$$

The field u_i^s is the scattered field which we want to determine at distances far from the flaw. It is straightforward to show that as $r \to \infty$

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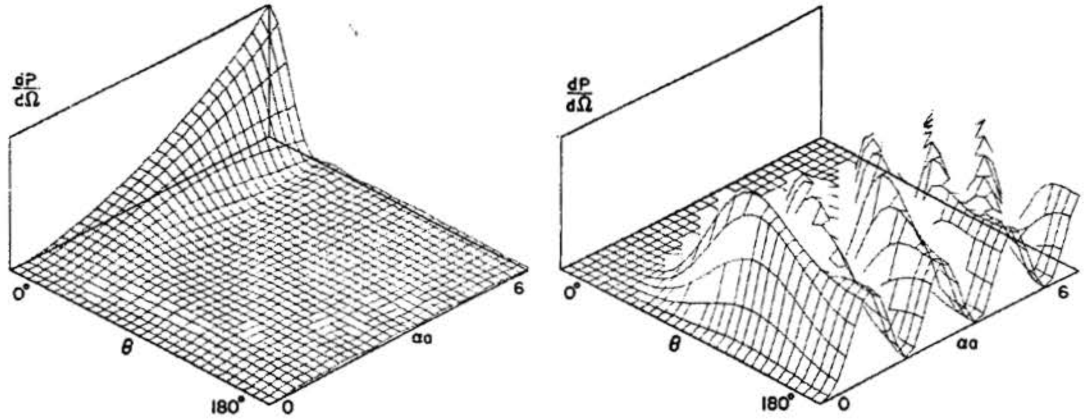


Exact

Born

Figure 1. The complete differential cross sections, based on the exact calculation and the Born approximation, for the scattering of a longitudinally polarized plane wave from a spherical cavity in Ti.

$$u_i^s = \frac{e^{i\alpha r}}{r} A_i + \frac{e^{i\beta r}}{r} B_i . \tag{7}$$

The vectors A_i and B_i are called the scattered amplitudes. A_i is parallel to the longitudinal direction while B_i is perpendicular.

The longitudinal scattered amplitude $A_i(\theta,\varphi)$ is

$$A_i(\theta,\varphi) = \frac{1}{4\pi\rho\omega^2}\hat{r}_i\,\alpha^2\Big[\delta\rho\omega^2\int_R d\underline{r}'\,\hat{r}_j u_j\, e^{-i\underline{\alpha}\cdot\underline{r}'} + i\alpha\,\hat{r}_j\hat{r}_k\,\delta C_{jklm}\int_R d\underline{r}'\,u_{l,m'}\,e^{-i\underline{\alpha}\cdot\underline{r}'}\Big], \tag{8}$$

and the transverse scattered amplitude $B_i(\theta,\varphi)$ is

$$B_i(\theta,\varphi) = \frac{1}{4\pi\rho\omega^2}\,\beta^2(\delta_{ij}-\hat{r}_i\hat{r}_j)\Big[\delta\rho\omega^2\int_R d\underline{r}'u_j\,e^{-i\underline{\beta}\cdot\underline{r}'} + i\beta\,\hat{r}_k\,\delta C_{jklm}\int_R d\underline{r}'\,u_{l,m'}\,e^{-i\underline{\beta}\cdot\underline{r}'}\Big]. \tag{9}$$

From these expressions, one easily sees that the scattered amplitudes depend on the displacement and strain fields inside the flaw. An adequate determination of those fields is the central difficulty of the problem; in general, they can only be approximated.

If the displacement field associated with the incident power is a plane wave traveling in the z-direction and having the form

$$u_i^o = a_i\,e^{i(\alpha z - \omega t)} + b_i\,e^{i(\beta z - \omega t)} \tag{10}$$

where a_i and b_i are the vector amplitudes of the longitudinal and transverse components of the wave, then we can show that the differential cross section equals

$$\frac{dP(\omega)}{d\Omega} = \frac{\alpha(\lambda+2\mu)\,|A_i|^2 + \beta\mu\,|B_i|^2}{\alpha(\lambda+2\mu)\,|a_i|^2 + \beta\mu\,|b_i|^2}. \tag{11}$$

The cases we present below are for an incident longitudinal plane wave of unit amplitude; hence,

$$\frac{dP(\omega)}{d\Omega} = |A_i|^2 + \frac{\alpha}{\beta}\,|B_i|^2. \tag{12}$$

It is evident that the part of the incident wave, polarized longitudinally, appears in the scattered power polarized transversely. The appearance of the second mode in the scattered power is the well-known mode conversion. Below we refer to $|A_i|^2$ as the longitudinal differential cross section $dP_L/d\Omega$ and $|B_i|^2$ as the transverse differential cross section $dP_T/d\Omega$. We also refer to the cross section in (14) as the complete differential cross section.

The Born Approximation

Integral equations generally have iterative solutions. A simple approximation to the solution is to keep the first term in the iteration. In quantum mechanics, this approximation is called the first Born approximation. Physically, in the present case, the displacement and strain fields in (8) and (9) are approximated by the displacement and strain fields associated with the incident wave; that is, the fields that would be there if not for the flaw. Mathematically, we substitute u_i^o and $u_{i,j}^o$ for u_i and $u_{i,j}$ in (8) and (9). For an incident longitudinally polarized plane wave of unit amplitude, we find that

$$A_i(\theta,\varphi) = (4\pi)^{-1}\,\alpha^2\,\hat{r}_i S_\alpha(\alpha;\theta,\varphi) \times \Big[\frac{\delta\rho}{\rho}\cos\theta - \frac{\delta\lambda + 2\delta\mu\cos^2\theta}{\lambda+2\mu}\Big] \tag{13}$$

and

$$B_i(\theta,\varphi) = (4\pi)^{-1}\,\beta^2\,\hat{\theta}_i S_\alpha(\beta;\theta,\varphi) \times \Big[\frac{2\alpha\,\delta\mu\cos\theta\sin\theta}{\mu\beta} - \frac{\delta\rho\sin\theta}{\rho}\Big]. \tag{14}$$

The quantity $S_p(q;\theta,\varphi)$ is called the shape factor as it is the Fourier transform of the shape of the flaw:

$$S_p(q;\theta,\varphi) = \int_R d\underline{r}'\,e^{i(p\hat{z} - q\hat{r})\cdot\hat{r}'}. \tag{15}$$

Denoting Δk as $p\hat{z} - q\hat{r}$, for a sphere of a radius a, we find that

$$S = 4\pi a^3\,\frac{\sin(a\Delta k) - a\Delta k\cos(a\Delta k)}{(a\Delta k)^3}. \tag{16}$$

The use of (13), (14), and (16) in (12) yields the

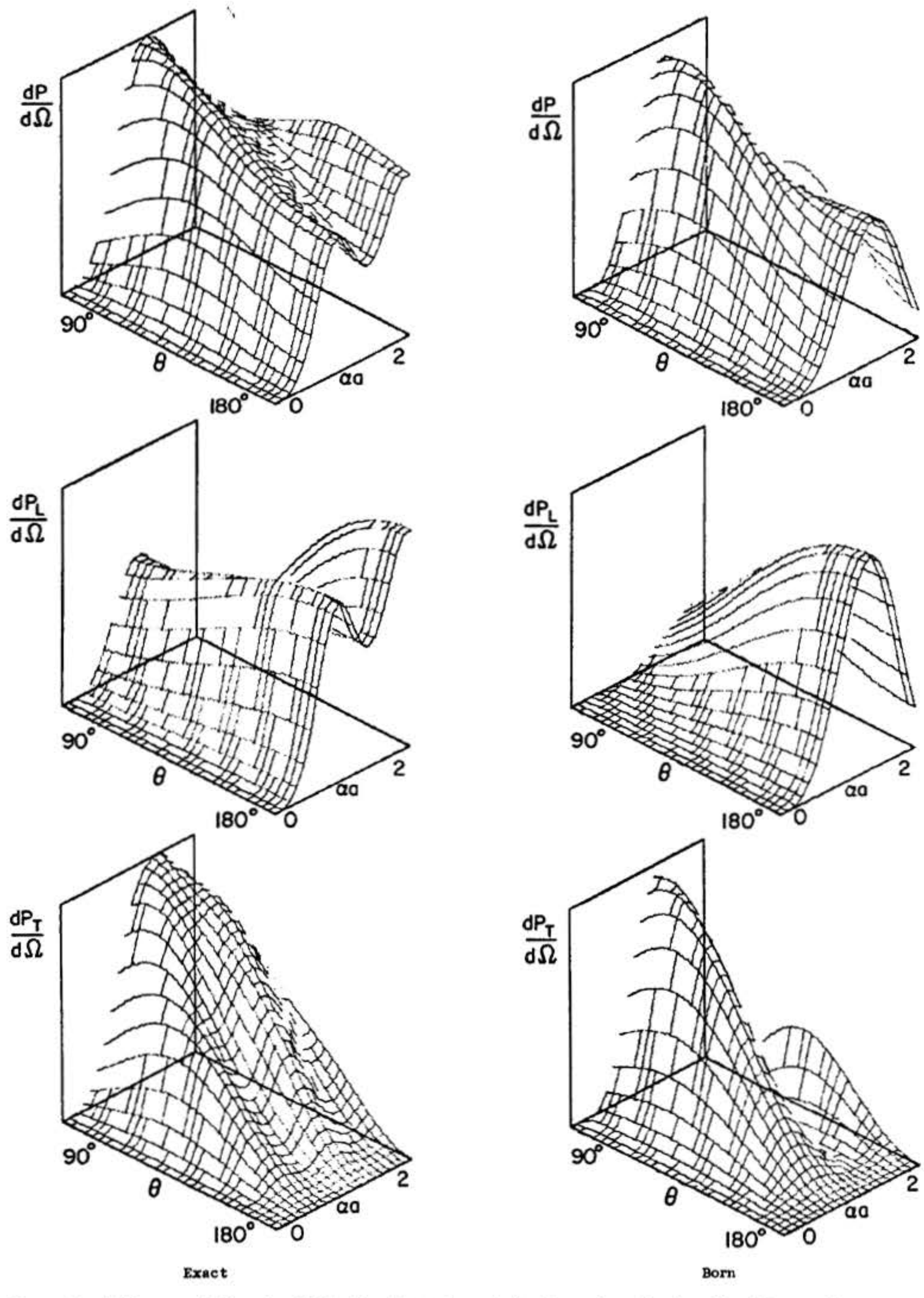
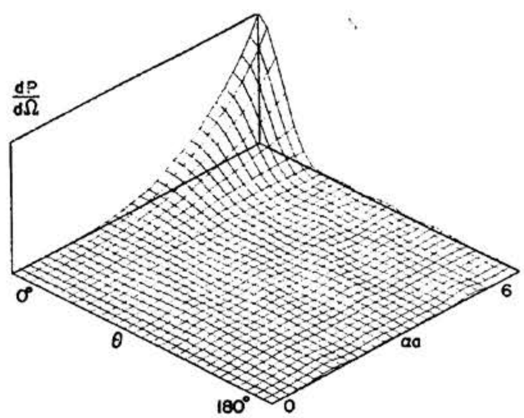


Figure 2. A blow-up of Fig. 1, eliminating large αa and the forward scattering directions. The separate longitudinal and transverse differential cross sections are shown in addition to the complete differential cross section.



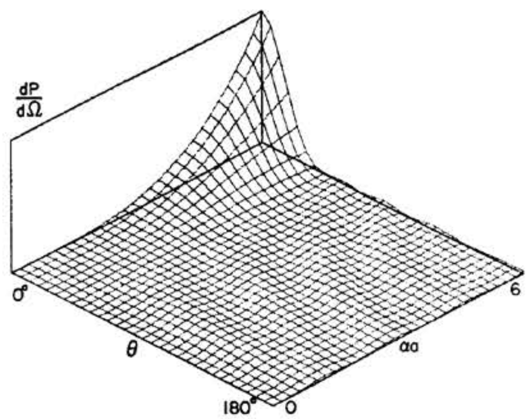


Figure 3. The complete differential cross sections, based on the exact calculation and the Born approximation, for the scattering of a longitudinally polarized plane wave from an $A\ell$ sphere embedded in Ti.

complete differential cross section in the first Born approximation for the scattering of an incident longitudinally polarized plane from a spherical flaw - a cavity or an elastic inclusion.

Results and Conclusions

Exact solutions exist for the scattering of an incident longitudinal plane wave from a spherical elastic inclusion and cavity.[3] These exact solutions are easily adaptable to numerical analysis on the computer. We use the exact, computer solutions to calibrate the Born approximation.

In Fig. 1 we show the comparison of the complete differential cross section of the exact solution with that of the Born approximation for scattering from a spherical cavity in Ti. It is important to note that Fig. 1 and subsequent figures are normalized so that the maximum height of each surface is the same. Thus, only the shapes of the surfaces are relevant. Over the entire range of αa and θ the agreement between the two calculations is rather poor, particularly in the forward ($\theta = 0^{\circ}$) scattering direction. Eliminating from consideration the forward scattering directions and looking at fairly small values αa, we see in Fig. 2 greatly improved agreement between the calculations. In this figure we also partition the complete differential cross section into the separate longitudinal and transverse contributions.

When the cavity is replaced by an $A\ell$ sphere, the agreement between the calculations is striking. This result is shown in Fig. 3. We expect this improved agreement since the properties of an $A\ell$ sphere are closer to those of the host material than are the properties of a cavity: The scattering is weaker and consequently more suitable to perturbation analysis. The excellent agreement between the calculations over nearly the entire range of the calculation is unexpected.

From Figs. 1, 2, and 3 we conclude that the Born approximation gives a useful description of the scattering of a longitudinally polarized plane wave from a spherical cavity and elastic inclusion, at least when $\alpha a \leq 1$ and when the scattering angle is restricted to backward directions. The principal defect of the Born approximation is an inadequate description of forward scattering for $\alpha a > 1$.

We believe that the Born approximation can be useful in some NDE applications, especially ones operating in a reflection mode. Although at present our statement is supported by calculations for the case of a sphere, we believe our conclusions will remain firm for nonspherical flaw geometries when $\alpha\ell < 1$ where ℓ is some minimal, but characteristic, length of the flaw. One of the most attractive features of the Born approximation is the ease of the inclusion of nonspherical geometries: All that is required is a different shape factor. This function is easily evaluated for many relevant shapes.

Elsewhere we will publish a more detailed description of the features of the integral equation and a more extensive comparison of the exact calculation and the Born approximation.[2] Included will be the case of the scattering of an incident transversely polarized plane wave from a spherical cavity and elastic inclusion.

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3. C. F. Ying and R. Truell, J. Appl. Phys. 27, 1086 (1956).

DISCUSSION

DR. BRUCE THOMPSON (Rockwell International Science Center): I'd like to ask the first question. How are these data normalized with respect to one another?

DR. JIM GUBERNATIS (Cornell University): The way I understand what the program does is that over the range which one has plotted, it takes the maximum and that sets the vertical scale.

DR. BRUCE THOMPSON: Okay. We have a few minutes for a couple of questions. Henry?

DR. HENRY BERTONI (Polytechnical Institute of New York): If you use the static solution instead of the unperturbed solution for small ka, you think you get the forward scattering correctly?

PROF. KRUMHANSL: Yes, you do for the case of the sphere. Sometime ago Mow, and Knopoff in a geophysical application showed that this is fine for the longitudinal wave, the longitudinal wave corresponding to uni-axial stress situation. Then what you do is put in for the u input the static solution for the imbedded cavity. That's why I said Eshelby solutions can be used.

DR. BERTONI: Do you expect that would also be good for all the other shear waves also?

PROF. KRUMHANSL: Yes.

DR. HARRY F. TIERSTEN (Rensselaer Polytechnical Institute): Are there any restrictions on the shape of the object?

PROF. KRUMHANSL: Not in principle.

DR. TIERSTEN: I ask that because it seems to me you could have various shaped objects that might give you the same integration where it would give you somewhat different fields.

PROF. KRUMHANSL: That relates to a question someone asked before as to whether the so-called inverse problem is unique. That is, formally, an unanswered question. In one sense the asymptotic scattered field is a Fourier transform with the asymptotic form of the Green's function of the defect material properties within the flaw region. Now, whether there's enough information in that transform, taking into account the vector components of the field, to uniquely determine either the shape or the $\Delta \rho$, is not well understood yet.

DR. TIERSTEN: So, in other words, you don't really know for sure, for any shape, what it really means relative to the exact solution, if someone could solve the full boundary problem?

PROF. KRUMHANSL: Just now I don't, but in the electromagnetic case, the geophysicists have approximated solutions of the inverse problem, subject to things you have to put in physically.

MR. DICK REYNOLDS (Advanced Research Projects Agency): You said, I believe, that there was no limitation on the shape of the defect that you could look at. Do you have in mind ultimately the case of the nearly flat crack with a sharp edge?

PROF. KRUMHANSL: Yes. Yes. We'll try to approach it and we're hoping for some help. If somebody will do the elipsoidal exact solution which we always want in a calibation solution, then we'll crack out our approximate calculation against it.

DR. BRUCE THOMPSON: Gordon, you had your hand up.

PROF. GORDON KINO (Stanford University): I have two questions. One is, I can see from one respect, with the Born approximation, to possibly do reasonably well in certain regions of the scattering angle, but I can't see how you can expect with a sharp discontinuity to get the amplitude right. I mean, just compared to the electromagnetic case with a sharp change in the dielectric constants, to get the amplitude of the field inside is very difficult.

PROF. KRUMHANSL: Well, we're not there yet, but if I look at the horizon with loving eyes, then I can believe that what you can do is something that has been done in some cases of propagation in composite media, that I have been involved with. You can take a linear combination of some more or less smooth solution and the singular solution of a static problem for an edge or for a corner, and then, since this is a self adjoint integral equation, you vary those linear combinations and a computer loves to do that.

PROF. KINO: That was the second question I was going to ask you. Can you put this into a variational form?

PROF. KRUMHANSL: Yes, you can. We haven't done it.